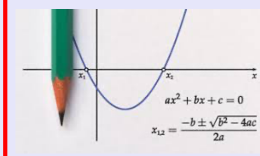
Math 125

Spring 2022

Lecture 13



Class QZ 10

I asked one student earlier to share her reaction about something.

1) what was her name?

Maria

2) Reaction was about what?

for students join the meeting late.

Matrix → Rectangular array or table

Matrix Notation $A = \begin{bmatrix} & & \end{bmatrix}$

A matrix has m rows and n columns.

Size of a matrix is $m \times n$.

Ex: $A = \begin{bmatrix} 1 & 2 & 5 \\ 3 & -1 & 2 \end{bmatrix}$ $B = \begin{bmatrix} 1 & -1 & 4 \\ 2 & 3 & 0 \\ -1 & 5 & 2 \end{bmatrix}$

2×3 Rows Columns

3×3 Rows Columns

$C = \begin{bmatrix} 2 \\ 0 \\ -3 \\ 5 \end{bmatrix}$ Column matrix

4×1 matrix

$D = \begin{bmatrix} 1 & -1 & 2 & -2 & 0 \end{bmatrix}$

Row matrix
1×5

Square Matrix ⇒ # Rows = # Columns

$A = \begin{bmatrix} 2 & -3 \\ 1 & 4 \end{bmatrix}$

2×2 Matrix

$B = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 5 & 6 & 7 \\ 0 & 0 & 8 & 9 \\ 0 & 0 & 0 & 10 \end{bmatrix}$

4×4 matrix

There is a number associated with every square matrix, and it is called its determinant.

Notation $|A|$ It could be $+$, $-$, or 0.

$\det(A)$

$A = \begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix}$ $\quad |A| = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$

$= a_1 b_2 - a_2 b_1$

Evaluate $\begin{vmatrix} 2 & 3 \\ 1 & 5 \end{vmatrix} = 2(5) - 1(3)$

$= 10 - 3 = \boxed{7}$ ✓ Positive

Evaluate $\begin{vmatrix} 3 & 6 \\ 2 & -3 \end{vmatrix} = 3(-3) - 2(6)$

$= -9 - 12 = \boxed{-21}$ ✓ Negative

Evaluate $\begin{vmatrix} 2 & -5 \\ -6 & 15 \end{vmatrix} = 2(15) - (-6)(-5) = 30 - 30$

Zero $= \boxed{0}$ ✓

Cramer's Rule:

$$\begin{cases} a_1 x + b_1 y = c_1 \\ a_2 x + b_2 y = c_2 \end{cases}$$

$D = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$

$D_x = \begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}$

$D_y = \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}$

$x = \dfrac{D_x}{D} \qquad y = \dfrac{D_y}{D}$

If $D \neq 0$

Solve by Cramer's Rule

$$\begin{cases} 2x + y = 5 \\ x - 2y = -5 \end{cases}$$

$$D = \begin{vmatrix} 2 & 1 \\ 1 & -2 \end{vmatrix} = 2(-2) - 1(1) = \boxed{-5}$$

$$D_x = \begin{vmatrix} 5 & 1 \\ -5 & -2 \end{vmatrix} = 5(-2) - (-5)(1) = \boxed{-5}$$

$$D_y = \begin{vmatrix} 2 & 5 \\ 1 & -5 \end{vmatrix} = 2(-5) - 1(5) = \boxed{-15}$$

$$x = \frac{D_x}{D} = \frac{-5}{-5} = 1$$

$$y = \frac{D_y}{D} = \frac{-15}{-5} = 3$$

$\boxed{\text{Final Ans } (1,3)}$ Soln. Set $\{(1,3)\}$

Solve by Cramer's rule:

$$\begin{cases} 3x + 2y = 12 \\ 2x + 3y = 13 \end{cases}$$

$$D = \begin{vmatrix} 3 & 2 \\ 2 & 3 \end{vmatrix} = 3(3) - 2(2) = \boxed{5}$$

$$D_x = \begin{vmatrix} 12 & 2 \\ 13 & 3 \end{vmatrix} = 12(3) - 13(2) = \boxed{10}$$

$$D_y = \begin{vmatrix} 3 & 12 \\ 2 & 13 \end{vmatrix} = 3(13) - 2(12) = \boxed{15}$$

$$x = \frac{D_x}{D} = \frac{10}{5} = 2$$

$$y = \frac{D_y}{D} = \frac{15}{5} = 3$$

$\boxed{\text{Final Ans } (2,3)}$ Soln. Set $\{(2,3)\}$

How to evaluate a 3x3 determinant:

Expand by first row

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = a_1\begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} \underset{\uparrow \text{Always}}{-} b_1\begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} + c_1\begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix}$$

Evaluate

$$\begin{vmatrix} 1 & 2 & 4 \\ 2 & 3 & 6 \\ 0 & 1 & 8 \end{vmatrix} = 1\begin{vmatrix} 3 & 6 \\ 1 & 8 \end{vmatrix} - 2\begin{vmatrix} 2 & 6 \\ 0 & 8 \end{vmatrix} + 4\begin{vmatrix} 2 & 3 \\ 0 & 1 \end{vmatrix}$$

$$= 1(3(8)-1(6)) - 2(2\cdot 8 - 0\cdot 6) + 4(2\cdot 1 - 0\cdot 3)$$

$$= 1(24-6) - 2(16-0) + 4(2-0)$$

$$= 18 - 32 + 8 = 26 - 32$$

$$= \boxed{-6}$$

Evaluate

$$\begin{vmatrix} 2 & -3 & 0 \\ 1 & 5 & 3 \\ -1 & 2 & -8 \end{vmatrix} = 2\begin{vmatrix} 5 & 3 \\ 2 & -8 \end{vmatrix} \underset{\text{Always}}{-} (-3)\begin{vmatrix} 1 & 3 \\ -1 & -8 \end{vmatrix} + 0\begin{vmatrix} 1 & 5 \\ -1 & 2 \end{vmatrix}$$

$$= 2(-40 - 6) + 3(-8 - (-3)) + 0 \cdot \text{who cares!}$$

$$= 2(-46) + 3(-5) = -92 - 15 = \boxed{-107}$$

Evaluate by expanding about the first row:

$$\begin{vmatrix} 1 & 2 & -3 \\ 3 & 1 & 4 \\ 4 & 3 & 1 \end{vmatrix} = 1\begin{vmatrix} 1 & 4 \\ 3 & 1 \end{vmatrix} - 2\begin{vmatrix} 3 & 4 \\ 4 & 1 \end{vmatrix} + (-3)\begin{vmatrix} 3 & 1 \\ 4 & 3 \end{vmatrix}$$

$$= 1(1-12) - 2(3-16) - 3(9-4)$$

$$= -11 + 26 - 15 = 15 - 15 = \boxed{0}$$

Evaluate $\begin{vmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{vmatrix}$

$$= 1\begin{vmatrix} 4 & 5 \\ 0 & 6 \end{vmatrix} - 2\begin{vmatrix} 0 & 5 \\ 0 & 6 \end{vmatrix} + 3\begin{vmatrix} 0 & 4 \\ 0 & 0 \end{vmatrix}$$

$$= 1(24-0) - 2(0-0) + 3(0-0)$$

$$= 24 + 0 + 0 = \boxed{24}$$

Cramer's Rule for

$$\begin{cases} a_1 x + b_1 y + c_1 z = d_1 \\ a_2 x + b_2 y + c_2 z = d_2 \\ a_3 x + b_3 y + c_3 z = d_3 \end{cases}$$

$$D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$$D_x = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}$$

$$D_y = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}$$

$$D_z = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}$$

$x = \frac{D_x}{D}$

$y = \frac{D_y}{D}$ $\quad D \neq 0 \quad (x, y, z), \ \{(x, y, z)\}$

$z = \frac{D_z}{D}$

Solve by Cramer's Rule:

$$\begin{cases} x + y + z = 6 \\ 2x - y = 0 \\ 3y - 2z = 0 \end{cases}$$

$$D = \begin{vmatrix} 1 & 1 & 1 \\ 2 & -1 & 0 \\ 0 & 3 & -2 \end{vmatrix}$$

$$D_x = \begin{vmatrix} 6 & 1 & 1 \\ 0 & -1 & 0 \\ 0 & 3 & -2 \end{vmatrix} \qquad D_y = \begin{vmatrix} 1 & 6 & 1 \\ 2 & 0 & 0 \\ 0 & 0 & -2 \end{vmatrix}$$

$$D_z = \begin{vmatrix} 1 & 1 & 6 \\ 2 & -1 & 0 \\ 0 & 3 & 0 \end{vmatrix}$$

$$D = \begin{vmatrix} 1 & 1 & 1 \\ 2 & -1 & 0 \\ 0 & 3 & -2 \end{vmatrix} = 1\begin{vmatrix} -1 & 0 \\ 3 & -2 \end{vmatrix} - 1\begin{vmatrix} 2 & 0 \\ 0 & -2 \end{vmatrix} + 1\begin{vmatrix} 2 & -1 \\ 0 & 3 \end{vmatrix}$$

$$= 1(2-0) - 1(-4-0) + 1(6-0)$$

$$= 2 + 4 + 6 = \boxed{12}$$

$$D_x = \begin{vmatrix} 6 & 1 & 1 \\ 0 & -1 & 0 \\ 0 & 3 & -2 \end{vmatrix} = 6\begin{vmatrix} -1 & 0 \\ 3 & -2 \end{vmatrix} - 1\begin{vmatrix} 0 & 0 \\ 0 & -2 \end{vmatrix} + 1\begin{vmatrix} 0 & -1 \\ 0 & 3 \end{vmatrix}$$

$$= 6(2-0) - 1(0-0) + 1(0-0)$$

$$= 6 \cdot 2 - 0 + 0 = \boxed{12}$$

$$x = \frac{D_x}{D} = \frac{12}{12} = \boxed{1} \qquad y = \frac{D_y}{D} = \frac{24}{12} = 2$$

verify

$$z = \frac{D_z}{D} = \frac{36}{12} = 3$$

Final Ans: $\boxed{(1,2,3)} \rightarrow \{(1,2,3)\}$

Solve for $\boxed{y \text{ only}}$ by Cramer's rule

$$\begin{cases} 2x + y = 5 \\ y + z = 7 \\ 4x - z = 0 \end{cases}$$

$$y = \frac{D_y}{D} = \frac{6}{2} = \boxed{3}$$

$$D = \begin{vmatrix} 2 & 1 & 0 \\ 0 & 1 & 1 \\ 4 & 0 & -1 \end{vmatrix} = 2\begin{vmatrix} 1 & 1 \\ 0 & -1 \end{vmatrix} - 1\begin{vmatrix} 0 & 1 \\ 4 & -1 \end{vmatrix} + 0 \cdot \text{who cares!}$$

$$= 2(-1) - 1(-4) = -2 + 4 = \boxed{2}$$

$$D_y = \begin{vmatrix} 2 & 5 & 0 \\ 0 & 7 & 1 \\ 4 & 0 & -1 \end{vmatrix} = 2\begin{vmatrix} 7 & 1 \\ 0 & -1 \end{vmatrix} - 5\begin{vmatrix} 0 & 1 \\ 4 & -1 \end{vmatrix} + 0 \cdot \text{who cares!}$$

$$= 2(-7) - 5(-4) = \boxed{6}$$

Solve for (z only) using (Cramer's rule:)

$$\begin{cases} x \quad\quad - z = 4 \\ \quad y + z = 2 \\ x - y \quad\quad = -4 \end{cases}$$

$$z = \frac{D_z}{D} = \frac{-6}{2} = \boxed{-3}$$

$$D = \begin{vmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \\ 1 & -1 & 0 \end{vmatrix} = 1\begin{vmatrix} 1 & 1 \\ -1 & 0 \end{vmatrix} - 0 \cdot \begin{vmatrix} \text{who} \\ \text{cares!} \end{vmatrix} + (-1)\begin{vmatrix} 0 & 1 \\ 1 & -1 \end{vmatrix}$$

$$= 1(0+1) - 0 - 1(0-1)$$

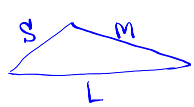
$$= 1 + 0 + 1 = \boxed{2}$$

$$D_z = \begin{vmatrix} 1 & 0 & 4 \\ 0 & 1 & 2 \\ 1 & -1 & -4 \end{vmatrix} = 1\begin{vmatrix} 1 & 2 \\ -1 & -4 \end{vmatrix} - 0\begin{vmatrix} \text{who} \\ \text{cares!} \end{vmatrix} + 4\begin{vmatrix} 0 & 1 \\ 1 & -1 \end{vmatrix}$$

$$= 1(-4+2) - 0 + 4(-1)$$

$$= -2 - 4 = \boxed{-6}$$

Perimeter of triangle ABC is 12cm.
twice the smallest side plus three times the longest side is 21 cm.
the difference of middle side and longest side is -1. find all three sides.



$$\begin{cases} S + M + L = 12 \\ 2S + 3L = 21 \\ M - L = -1 \end{cases}$$

$$\begin{cases} S + M + L = 12 \\ 2S \quad\quad + 3L = 21 \\ \quad M - L = -1 \end{cases}$$

$$\begin{cases} S + M + L = 12 \\ -1) \quad M - L = -1 \end{cases}$$

$$S \quad\quad + 2L = 13$$

$$-2\begin{cases} S + 2L = 13 \\ 2S + 3L = 21 \end{cases}$$

$$-L = -5 \qquad \boxed{L = 5}$$

$$S + 2(5) = 13$$

$$\boxed{S = 3}$$

$$M - 5 = -1$$

$$\boxed{M = 4}$$

three sides are

3cm, 4 cm, and 5cm.

I deposited \$5000 in 3 accounts for one Year, Simple interest.
one pays 2% , one pays 4% , and last one Pays 5% interest. Total interest made was \$214
money invest in 5% rate was \$600 less than three times the money invested at 4% account.
How much per account?

$x \rightarrow 2\%$ rate
$y \rightarrow 4\%$ rate
$z \rightarrow 5\%$ rate

$$100\begin{cases} x + y + z = 5000 \\ 2\%x + 4\%y + 5\%z = 214 \\ z = 3y - 600 \end{cases}$$

$$\begin{cases} x + y + z = 5000 \\ 2x + 4y + 5z = 21400 \\ -3y + z = -600 \end{cases}$$

Finish this by Thursday

I have 11 Coins.
Nickels, Dimes, Quarters only.
Total Value 90¢.
Dimes is three times # Quarters.
How many of each?

N → Nickels
D → Dimes
Q → Quarters

$$\div 5\begin{cases} N + D + Q = 11 \\ 5N + 10D + 25Q = 90 \\ D = 3Q \end{cases}$$

$$\begin{cases} N + D + Q = 11 \\ N + 2D + 5Q = 18 \\ D - 3Q = 0 \end{cases}$$

Finish this by Thursday

Graph of the equation $y = ax^2 + bx + c$

contains the points $(1,10), (-1,4)$, and $(2,19)$

Find the equation.

$(1,10) \Rightarrow 10 = a(1)^2 + b(1) + c \Rightarrow a+b+c = 10$

$(-1,4) \Rightarrow 4 = a(-1)^2 + b(-1) + c \Rightarrow a-b+c = 4$

$(2,19) \Rightarrow 19 = a(2)^2 + b(2) + c \Rightarrow 4a+2b+c = 19$

You should Solve by Thursday

Find a, b, and c.

Class QZ 11

Solve by addition method:

$$\begin{cases} 2x - 3y = -1 & \times 2 \\ 3x + 2y = 5 & \times 3 \end{cases} \Rightarrow \begin{cases} 4x - 6y = -2 \\ 9x + 6y = 15 \end{cases}$$

$13x = 13$ $\quad \boxed{x = 1}$

$2(1) - 3y = -1 \quad -3y = -3$ $\quad \boxed{y = 1}$

$(1,1), \{(1,1)\}$